2015

# Chapter 4
# The centroid and center of gravity

Shopra engeneering
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# Chapter 4
# centroid

مركز الثقل هو نقطة تأثير الوزن

ويختلف المركز الهندسي عن مركز الثقل في حالة ما كانت الكتلة غير موزعة بثبات علي الجسم "اي ان الكثافة متغيرة"

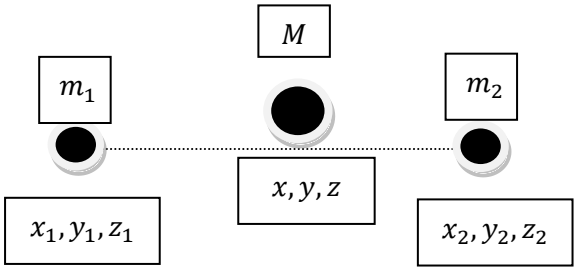
The centroid different from center of gravity in the case of the mass doesn't distribute by equal on the body (i.e. the density variable).

## THE CENTER OF GRAVITY FOR TWO MASSES

$$\bar{x} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

$$\bar{y} = \frac{y_1 m_1 + y_2 m_2}{m_1 + m_2}$$

$$\bar{z} = \frac{z_1 m_1 + z_2 m_2}{m_1 + m_2}$$




## THE CENTER OF GRAVITY FOR FINITE NUMBER OF MASSES

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i m_i}{\sum m_i}, \qquad \bar{y} = \frac{\sum_{i=1}^{n} y_i m_i}{\sum m_i}, \qquad \bar{z} = \frac{\sum_{i=1}^{n} z_i m_i}{\sum m_i}$$

## THE CENTER OF GRAVITY FOR RIGID BODY

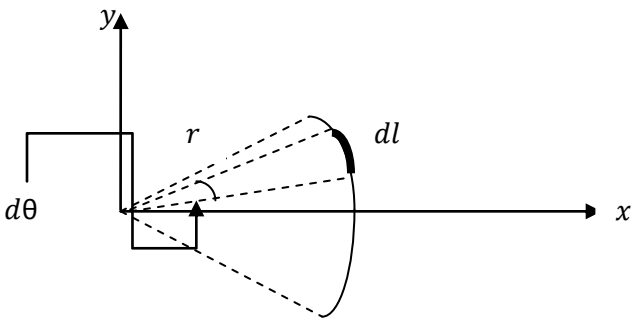
$$\bar{x} = \frac{\int \tilde{x} dm}{\int dm} \quad , \qquad \bar{y} = \frac{\int \tilde{y} dm}{\int dm} \quad , \qquad \bar{z} = \frac{\int \tilde{z} dm}{\int dm}$$

Where $\tilde{x}, \tilde{y}, \tilde{z}$ the centroid of the element or lamina

Example 1

Find the centroid for the arc of circle of radius r as shown in figure.

Solution

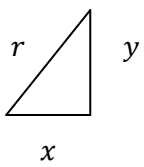



$$\bar{x} = \frac{\int \tilde{x} dl}{\int dl} = \frac{\int_{-\alpha}^{\alpha} x \, r d\theta}{\int_{-\alpha}^{\alpha} r d\theta} = \frac{\int_{-\alpha}^{\alpha} r \cos\theta \, r d\theta}{\int_{-\alpha}^{\alpha} r d\theta}$$

$$= \frac{[r \sin\theta]^{\alpha}_{-\alpha}}{[\theta]^{\alpha}_{-\alpha}} = \frac{r \sin\alpha}{\alpha}$$

$$dl = r d\theta$$

$$x = r\cos\theta$$



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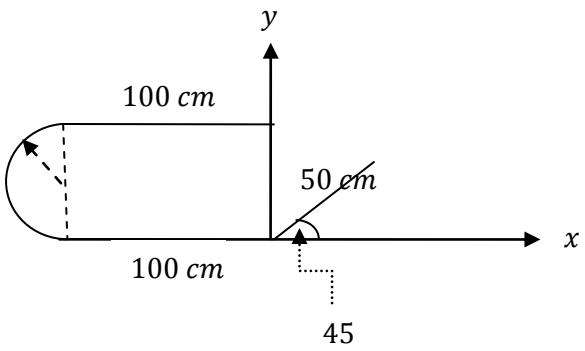
Example 2

Find the centroid for the shown wire.




Solution

| Part | L | $x$ | $y$ |
|---|---|---|---|
| 1 | 100 | -25 | 150 |
| 2 | $\pi$ (75) | -100 - $\frac{2(75)}{\pi}$ | 75 |
| 3 | 100 | -25 | 0 |
| 4 | 50 | 25 cos 45 | 25 sin 45 |

$$\bar{x} = \frac{x_1 l_1 + x_2 l_2 + x_3 l_3}{l_1 + l_2 + l_3} = -80.2$$

$$\bar{y} = \frac{y_1 l_1 + y_2 l_2 + y_3 l_3}{l_1 + l_2 + l_3} = 69.1$$

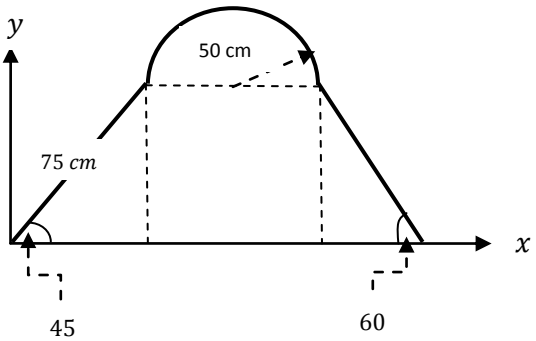
Example 3

Find the centroid for the shown wire.

Solution

75 sin 45= $L_3$ sin 60

$L_3$ =61.2




| part | L | $x$ | $y$ |
|---|---|---|---|
| 1 | 75 | $\frac{75}{2}$ cos 45 | $\frac{75}{2}$ sin 45 |
| 2 | $\pi$ (50) | 75 cos 45+50 | 75 sin 45+- $\frac{2(50)}{\pi}$ |
| 3 | 61.2 | 75 cos 45+100 +$\frac{61.2}{2}$ cos 60 | $\frac{61.2}{2}$ sin 60 |

$$\bar{x} = \frac{x_1 l_1 + x_2 l_2 + x_3 l_3}{l_1 + l_2 + l_3} =$$

$$\bar{y} = \frac{y_1 l_1 + y_2 l_2 + y_3 l_3}{l_1 + l_2 + l_3} =$$

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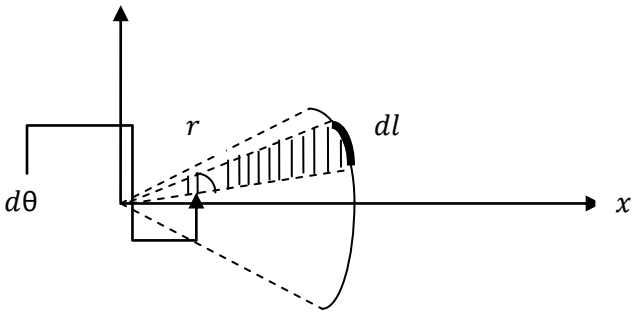
Example 3

Find the centroid for the sector of circle of radius r as shown in figure.

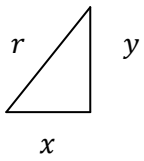



$$\bar{x} = \frac{\int \tilde{x} dA}{\int dA} = \frac{2\int_{-\alpha}^{\alpha} r\cos\theta \frac{1}{2} r\,(rd\theta)}{3\int_{-\alpha}^{\alpha} \frac{1}{2} r(rd\theta)} =$$

$$= \frac{2\int_{-\alpha}^{\alpha} r\,\cos\theta\, d\theta}{3\int_{-\alpha}^{\alpha} d\theta} = \frac{2[r\,sin\,\theta]^{\alpha}{}_{-\alpha}}{3[\theta]_{-\alpha}^{\alpha}} = \frac{2r\sin\alpha}{3\alpha}$$

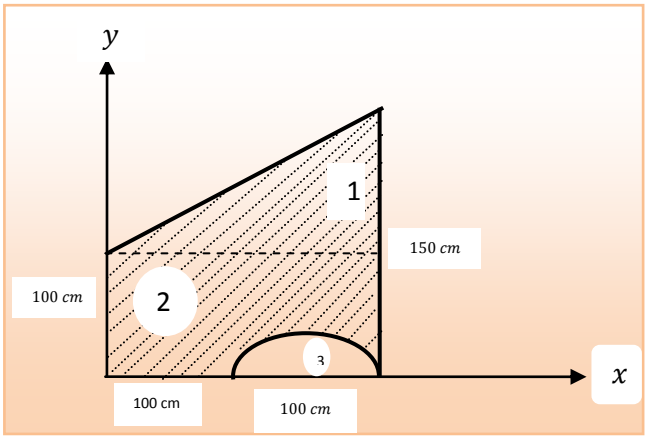
$$dA = \frac{1}{2} r dl = \frac{1}{2} r(rd\theta)$$

$$\tilde{x} = \frac{2}{3} r\cos\theta$$




Example 5

Find the centroid for the shown area




| part | $A$ | $x$ | $y$ |
|---|---|---|---|
| 1 | $\frac{1}{2} * 200 * 50$ | $\frac{2}{3}$ 200 | $100 + \frac{1}{3}(50)$ |
| 2 | $200 * 100$ | 100 | 50 |
| 3 | $-\frac{\pi * 50^2}{2}$ | 100+50 | $\frac{4(50)}{3\pi}$ |

$$\bar{x} = \frac{x_1A_1 + x_2A_2 + x_3A_3}{A_1 + A_2 + A_3} =$$

$$\bar{y} = \frac{y_1A_1 + y_2A_2 + y_3A_3}{A_1 + A_2 + A_3} =$$

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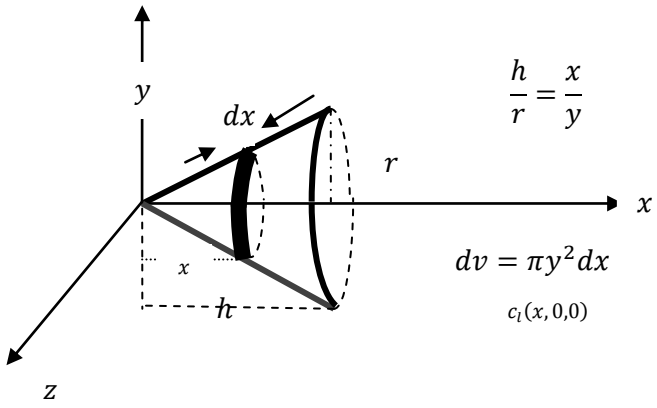
Example 6

Find the centroid for the solid cone shown in figure.




$$\bar{x}=\frac{\int \tilde{x}dv}{\int dv}=\frac{\int_0^h x(\pi y^2 dx)}{\int_0^h \pi y^2 dx}$$

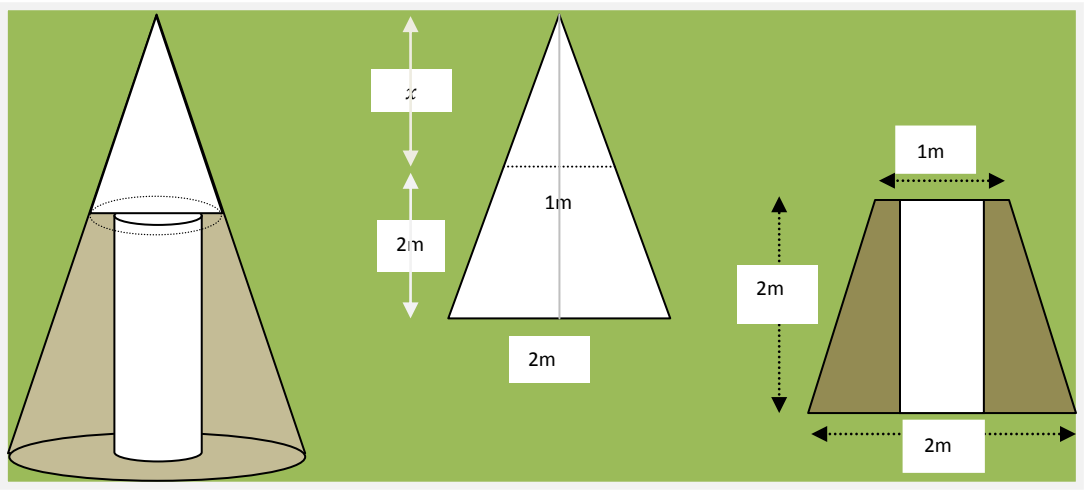
$$=\frac{\int_0^h x(\frac{r^2x^2}{h^2}\ dx)}{\int_0^h \frac{r^2x^2}{h^2}\ dx}$$

$$=\frac{\int_0^h x^3 dx}{\int_0^h x^2 dx}=\frac{[\frac{x^4}{4}]}{\frac{x^3}{3}}=\frac{3}{4}h$$

i.e. The centroid of the cone is $\frac{3}{4}h$ from the vertex and is equal $\frac{1}{4}h$ from the base

Example 7

Find the centroid for the frustum of solid cone that has an axial hole of 50 cm.




$$\frac{x}{x+2}=\frac{1}{2}$$

$$x=2$$

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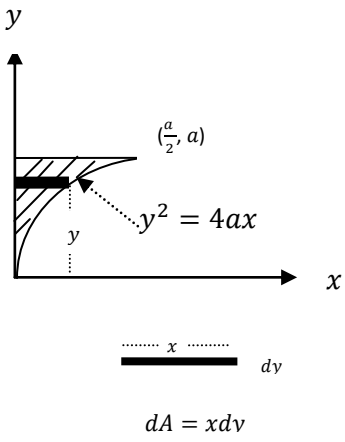
| part | $V$ | $z$ |
|---|---|---|
| 1 | $\frac{1}{3} * \pi * 1^2 * 4$ | $\frac{1}{4} * 4$ |
| 2 | $-\frac{1}{3} * \pi * (0.5)^2 * 2$ | $2+ \frac{1}{4} * 2$ |
| 3 | $-\pi * (0.25)^2 * 2$ | $\frac{1}{2} * 2$ |

$$\bar{z} = \frac{z_1 V_1 + z_2 V_2 + z_3 V_3}{V_1 + V_2 + V_3} =$$

Example 8

a- Find the centroid for the bounded area shown in fig. a.
b- If we revolved this area about y axis find the centroid for the resulting volume shown in fig. b.




$dA = xdv$

Fig.a

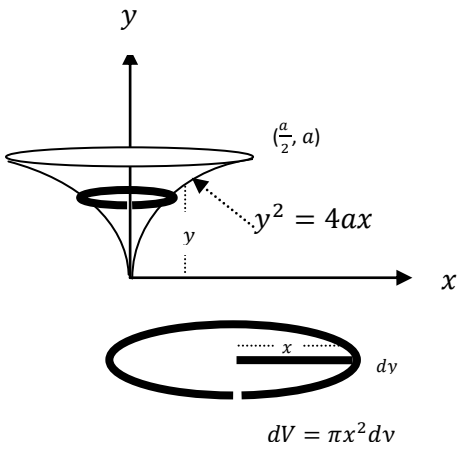



Fig.b

$dV = \pi x^2 dv$